EPFL

Systèmes dynamiques

Actionneurs et systèmes électromagnétiques

Equations des systèmes électromécaniques

$$u_j = R_j i_j + \frac{d\Psi_j}{dt}$$

(k circuits)

$$\Psi_j = \sum_{p=1}^{k} L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

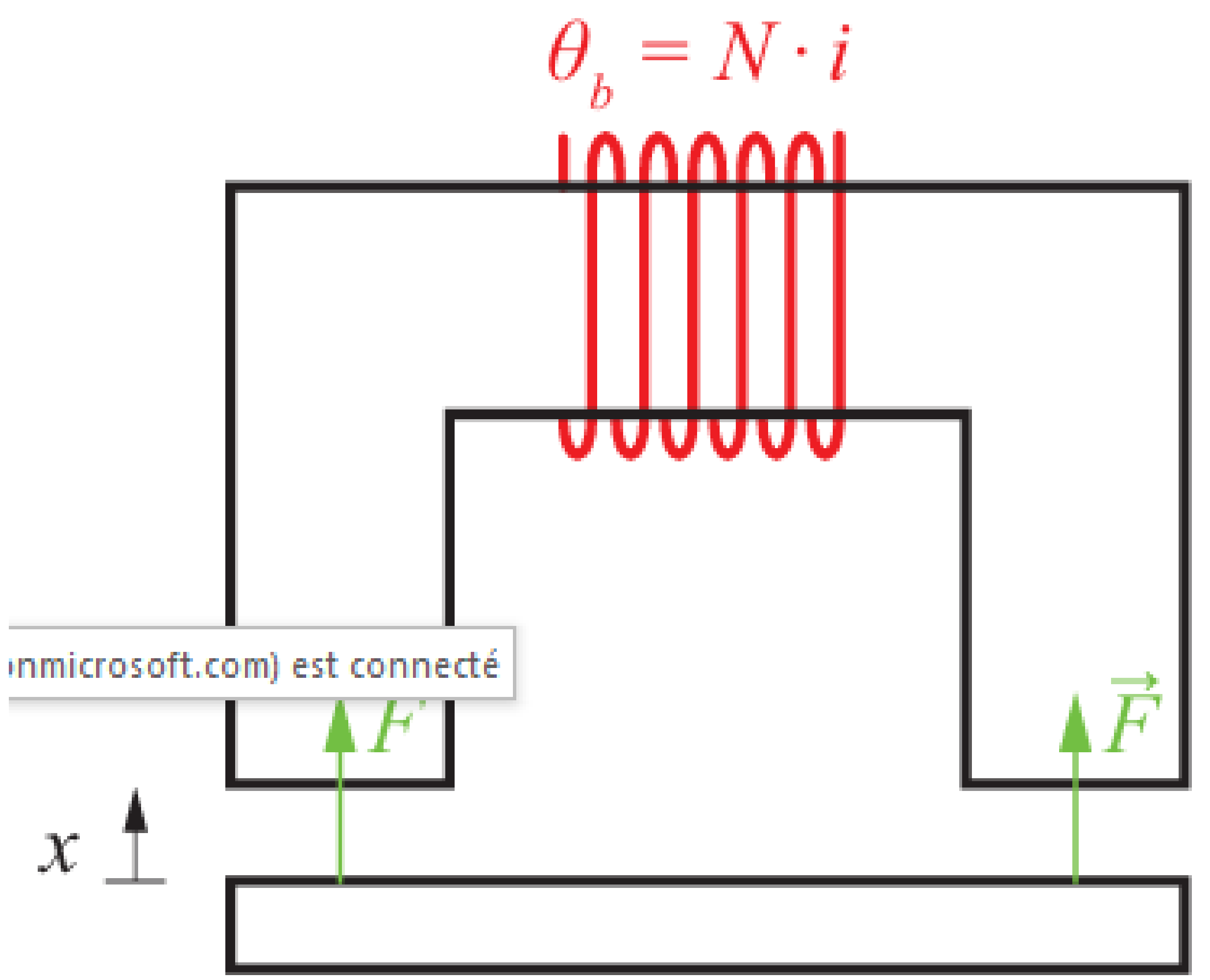
$$\sum M_m = J_m \frac{d^2 \alpha_m}{dt^2}$$

(n degrés de liberté)

$$F_m = \frac{1}{2} \sum_{j=1}^{k} \sum_{p=1}^{k} \frac{d\Lambda_{jp}}{dx_m} \Theta_j \Theta_p$$

$$M_m = \frac{1}{2} \sum_{j=1}^{k} \sum_{p=1}^{k} \frac{d\Lambda_{jp}}{d\alpha_m} \Theta_j \Theta_p$$

Exemple: système réluctant



Cas linéaire:

$$u = Ri + L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

$$u = Ri + L\frac{di}{dt} + \frac{2Fv}{i}$$

$$u_j = R_j i_j + \frac{d\Psi_j}{dt} \text{ avec } \Psi_j = \sum_{p=1}^{k} L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

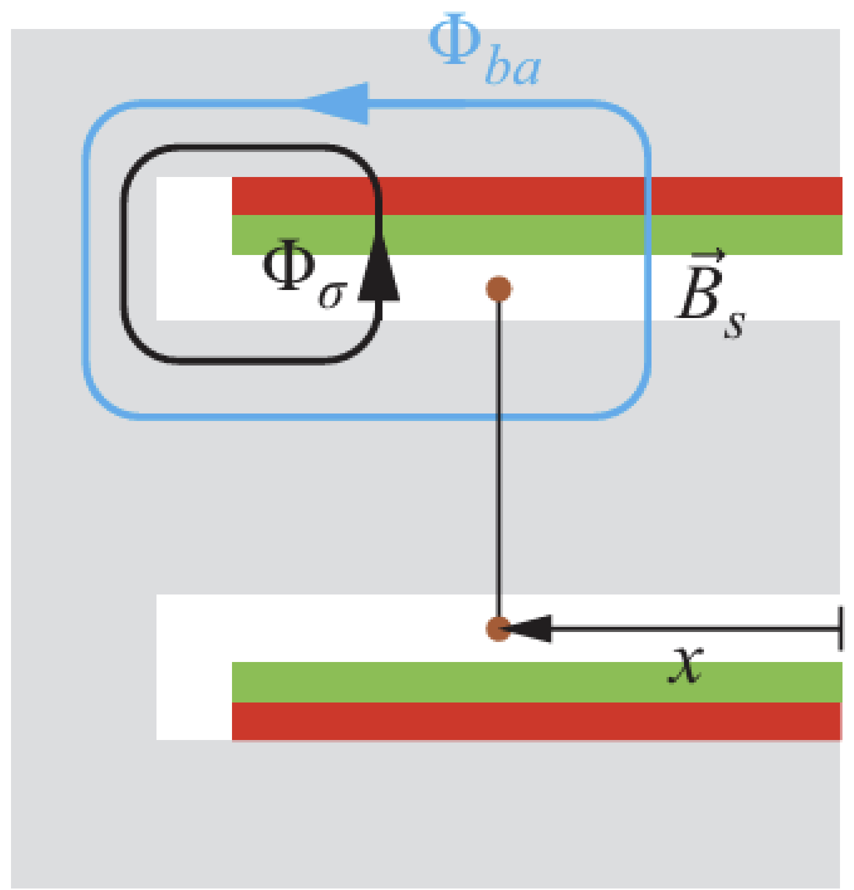
$$F_x = \frac{1}{2}\frac{d\Lambda_a}{dx}\Theta_a^2 + \frac{d\Lambda_{ab}}{dx}\Theta_a\Theta_b + \frac{1}{2}\frac{d\Lambda_b}{dx}\Theta_b^2$$

$$F_x = \frac{1}{2}\frac{d\Lambda_b}{dx}\Theta_b^2 = \frac{1}{2}\frac{dL}{dx}i^2 = m\frac{d^2x}{dt^2}$$

$$u = Ri + L\frac{di}{dt} + i\frac{dL}{dt} = Ri + L\frac{di}{dt} + i\frac{\partial L}{\partial x}\frac{dx}{dt} + i\frac{\partial L}{\partial i}\frac{di}{dt}$$

Exemple système réluctant (mesures)

Exemple: système polarisé



$$u_j = R_j i_j + \frac{d\Psi_j}{dt}$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

$$F_x = \frac{1}{2}\frac{d\Lambda_a}{dx}\Theta_a^2 + \frac{d\Lambda_{ab}}{dx}\Theta_a\Theta_b + \frac{1}{2}\frac{d\Lambda_b}{dx}\Theta_b^2$$

$$F_x = \frac{d\Lambda_{ab}}{dx}\Theta_a\Theta_b = \frac{d\Lambda_{ab}}{dx}\Theta_a N i$$

$$u = Ri + \frac{d\Psi}{dt}$$

avec:

$$\Psi = Li + \Psi_{ba} = Li + N\Lambda_{ba}\Theta_a$$

$$u = Ri + L\frac{di}{dt} + \frac{d\Psi_{ba}}{dt} = Ri + L\frac{di}{dt} + N\Theta_a\frac{d\Lambda_{ba}}{dx}\frac{dx}{dt}$$

Résolution

$$u = Ri + L\frac{di}{dt} + kv$$

$$F = m\frac{d^2x}{dt^2} = m\frac{dv}{dt}$$

$$F = ki$$

$$i = \frac{m}{k}\frac{dv}{dt}$$

$$u = R\frac{m}{k}\frac{dv}{dt} + L\frac{m}{k}\frac{d^2v}{dt^2} + kv$$

ou encore:

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{k^2}{mL}v = u\frac{k}{mL}$$